



U. Kivi

Gravitation

Exact calculation of Newton and Einstein theory

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Foreword

This is an amateur view of solving planet orbits according to classical mechanics and according to general theory of relativity. Classical theory is calculated in a reverse order. Starting point is an elliptic orbit and acceleration and force and time dependence are solved by demanding that acceleration is directed towards ellipse focal point. Results of classical theory are used as a starting point for relativistic orbit calculations.

In part two – General relativity – in chapter 2.6 ‘General solution’ a proposal is presented for the solution of space metric for the general case (arbitrary mass distribution in space). This is achieved as a generalization of Schwarzschild solution. You may give an almost mathematical justification for the method. The key question is whether integration over one ball when calculating Newton gravitation potential is somehow special compared to say integration over two balls (spaceship between Earth and Moon).

In chapter 2.7 ‘Free fall (one spatial dimension)’ free fall orbit is solved according to general relativity (velocity and acceleration as a function of position). The result is achieved by constructing Euler orbit equations in a different way than normally. The solution is verified by inserting into ‘normal’ Euler equations, which are fulfilled identically.

In chapter 2.8 ‘Planetary motion in a central field in a plane’ two-dimensional Euler equations are solved in the same way as one dimensional are solved in chapter 2.7. The results of chapters 2.7 and 2.8 are consistent in the sense that in the case of infinitely long major axis the two dimensional solution approaches the one dimensional solution. In addition both the two dimensional and one dimensional solutions approach the classical solutions as speed of light approaches infinity. Special feature in the new solution is that it gives a small extra constant term compared to former solutions. At Mercury orbit the extra constant term has a value that equals the usual constant term divided by about 40 million but closer to the sun the effect of the extra term increases.

At the end of the book there are some loose speculative chapters considering concrete ways how physics of general relativity might function.

This is an amateur view of general relativity. Hopefully book will be read also by general relativity professionals so that new solutions of chapters 2.6 'General solution', 2.7 'Free fall (one spatial dimension)' and 2.8 'Planetary motion in a central field in a plane' will be assessed.

Helsinki 1.1.2019 U.Kivi

Chapter 1: Classical gravitation

1.1 Planet orbit

When one creates a theory to describe planet orbit, one usually starts with a position dependent gravitation force. Force gives rise to acceleration and with the help of the acceleration one can calculate the orbit of the planet. Historically of course the events occurred in the reverse order. One first made observations of the orbits of the planets and based on the observations one created the theory to describe planet orbits. In this presentation of classical gravitation theory this historical order is followed. The results are of course the same as with the normal method, but in this way mathematics is easier.

One may consider planet orbit as a starting point for mechanics. The orbit may be described mathematically by giving the planet x and y coordinates as functions of time (we limit this description to two dimensions).

$$(1) \quad x = x(t)$$

$$(2) \quad y = y(t)$$

Motion may be described by quantities velocity, acceleration and force, which is thought to give reason for change of movement. Velocity is defined as the first time derivative of the position vector

$$(3) \quad v_x = \frac{dx(t)}{dt} = \dot{x}(t)$$

$$(4) \quad v_y = \frac{dy(t)}{dt} = \dot{y}(t)$$

and acceleration is defined to be the second time derivative of the position vector

$$(5) \quad a_x = \frac{d^2x(t)}{dt^2} = \ddot{x}(t)$$

$$(6) \quad a_y = \frac{d^2 y(t)}{dt^2} = \ddot{y}(t).$$

Force is defined to be the product of mass and acceleration.

$$(7) \quad F_x = ma_x$$

$$(8) \quad F_y = ma_y.$$

Galilei noticed that all objects accelerate in the gravitational field in the same way. In a sense Newton introduced quantity force in vain to describe gravitation. According to him, if one object has double mass compared to another object the gravitational force is double (gravitational mass), but the acceleration is the same, because double mass needs double force to achieve the same acceleration as the lighter object (inertial mass). If you weigh objects with masses 1 kg and 2 kg with a spring scale, you notice that the spring stretches more with 2 kg mass, so it is clear that gravitational force is greater for the 2 kg object, but acceleration may be the cause and force may be the result and the greater the mass of the object the greater the force it experiences. Einstein returned to this Galilei's original line of thinking, when he created the theory of general relativity.

Next we begin to create the theory of mechanics with the planet orbit as the starting point.

1.2 Elliptic orbit

According to observations planet moves around the sun along an elliptic orbit, and sun lies in the other focal point of the ellipse.

The general equation of an ellipse the center point of which lies in the origin and the major axis (length $2a$) of which lies along the x -axis and the minor axis (length $2b$) of which lies along the y -axis is

$$(1) \quad x = a \cos \Omega(t)$$

The presentation of the book leans heavily on mathematics, and good understanding of some parts of the book require skills in ground level university mathematics. You manage rather well also with high school mathematics and positive attitude (or maybe also with pure positive attitude). Both classical gravitation theory and general relativity theory are treated using easier mathematics than in standard presentations. The basic structure of the theories is presented clearly in the book. Some new results with relation to planet orbits are presented in the book.

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