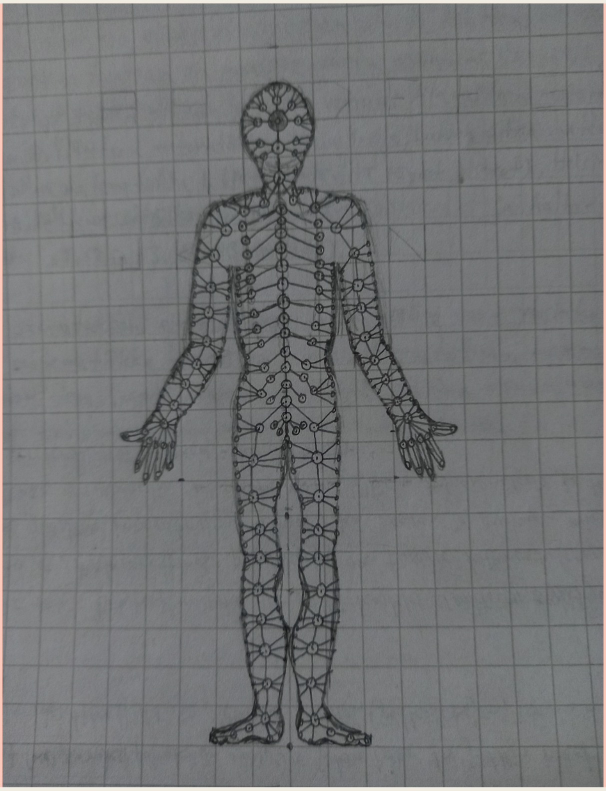
TREE GENERATION AND ENUMERATION

An extended model in graph theory



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For my mother

Contents

0.Introduction

I first started to apply creative thinking to mathematics when the year was about 2014. During that year it finally occurred to me that I wanted to make a book about this formal science, and especially the branch called graph theory. So, I finally created the book in the year 2015, after having made a couple of preceding manuscripts. This is the eighth book I have made about this subject so far, and the theory has become tremendously more sophisticated, when examining the first and last book in this sequence.

The advancement arc of the theory is the following:

Book Advancements

1 The creation of the base theory with symbols and primitively defined rules for vertex edge algebra, and a primitively explained process for the tree generation algorithm. No advanced aspects for tree enumeration yet. The generation case for free trees is not solved yet, and the formula (8) is not yet discovered.

2 The theory is now more formal, and there is some sort of an axiomatic system for the theory, although not a very developed one. The generation case for free trees has been partially solved, and the formula (8) has been discovered. Also, the explained process for the tree generation algorithm is more sophisticated, and the rules and symbols for vertex edge algebra are more clearly stated. Additionally, the enumeration tools are clearly more developed.

3 Includes the story for the birth of this theory and predicts the number of algorithm rows for the tree generation algorithm to be a low order polynomial. This prediction, however, turns out to be false, as the number is an exponentially growing function, more on this in the rooted tree enumeration examples.

4 The first breakthrough relative to this list, as the generation case for free trees is completely solved. Also, the rules and symbols are now more clearly stated than in the book 2. In addition, the enumeration tools are quite developed. There is now a clear, small section for isomorphisms that the free trees in vertex edge algebra introduce.

5 The clearly more developed version of the book 4, finally introducing the theorem of sum forms, although not in a very accurate form. Also has clearly stated rules and symbols for vertex edge algebra.

6 A clearly more accurate form of the theorem of sum forms than in the book 5, includes new branch types and an enumeration formula for rooted trees, hinting at a link between rooted trees and partitions.

7 The second breakthrough. Introduces the successor operator for rooted trees, showing a link between natural numbers and rooted trees. With a more accurate form of the theorem of sum forms than the book 6, paves the way for a formal system of tree generation. The isomorphism section is extended, experimental sections are added, and a list of possibly unsolved problems are stated in the end of the book.

8 The third breakthrough. The most formal and systematized book this far, has expanded the scope of tree generation to equations, introducing a new branch of mathematics. About double the size of the book 7, this book is a major addition to the theory of tree generation.

In the starting years of this journey, after having discussed with a couple of mathematicians, I finally got an idea to develop my theory even further by myself, especially thanks to the mathematicians in the University of Helsinki. Without my math teachers in high school and my uncle advising me to study a mathematical branch like engineering, I would not even have had the possibility of discovering this theory, so also a great thanks to them.

I think that intuition is a key element in discovering something new and building a theory. Imagination and creativity, when coupled with persistence and curiosity, work wonders in a mathematician's journey. Solving problems is an important part of developing a critical and flourishing mathematical mindset, but so is discovering something on your own, no matter if it has been discovered already. Solve mathematical problems, yes – but also create them! See if you can make a generalization of something if it's a special case and find special cases in generalizations! Possibilities are endless in mathematics, as are the cardinalities of real number lines between consecutive integers.

I am more of a theory builder than a problem solver. In mathematics, people tend to lean towards one of these sides in a spectrum, but nevertheless both are very important aspects regarding the advancement of science. You see, mathematics is a fundamental part of the scientific language and the models it constructs. Whether we shall see some grand unifying theories or breakthroughs in the future, ultimately may become the task for generations to come.

One interesting aspect of a sum of numbers is that it may depict many things at the same time.

For example, the sum 1+1 is

- a sum of numbers 1 and 1
- a sum whose result is two
- a partition of the number 2
- the tree where there is the root connected to an external vertex
- a binary operation between two Boolean values of 1
- the result of applying a successor operator to the single root tree
- a multiplication of one and two opened as a sum of ones

But is everything in mathematics possible to be expressed by numbers? Another question – now regarding this theory of mine – is if vertex edge algebra can be extended to graphs generally. My intuition says that it is not possible, maybe in some special cases, but probably not on the general case. Well, one special case is already done, namely, trees. What about the general case? I'll leave it to the next generation.

I believe in passing on knowledge to further generations, inspiring scientists and mathematicians of today and tomorrow. By making books about graph theory, I can make my vision happen and leave a mark in this world, and enrichen the minds of curious individuals.

The book Tree generation and enumeration: an extended model in graph theory, is about a formal system for tree generation, including such tree types as rooted trees, free trees, series-reduced rooted trees and series-reduced free trees. A link is made between positive integer partitions, natural numbers and trees, and the scope of tree generation is expanded to equations, a new branch in graph theory called advanced vertex edge algebra.

With multiple various examples and results, this model, or theory, can be an insightful experience, and can improve intuition and expertise especially in rooted tree generation. The main focus of the book is in tree generation, but there are quite many examples of tree enumeration as well.

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