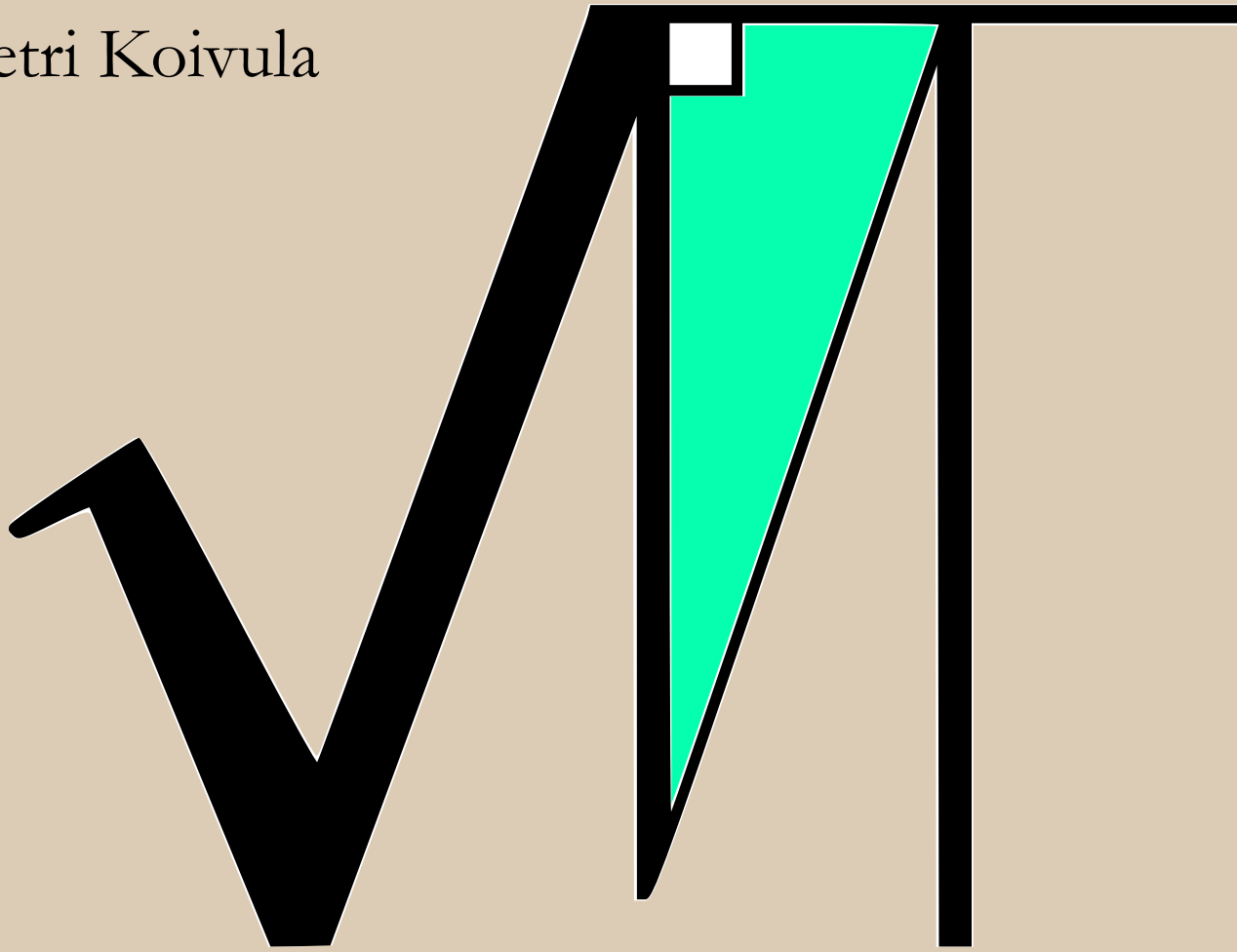
Petri Koivula



Imprint of Thought

Mathematics at the Borderland of Intuition and Structure

139 Problems with Solutions

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Cover design, layout, and illustrations: Petri Koivula

Publisher: BoD · Books on Demand, Mannerheimintie 12 B, 00100 Helsinki, bod@bod.fi
Printer: Libri Plureos GmbH, Friedensallee 273, 22763 Hamburg, Germany

ISBN 978-952-80-9590-3

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**Table 5 of the task classification presents the page numbers for each individual problem.*

Le savant digne de ce nom, le géomètre surtout, éprouve en face de son œuvre la même impression que l'artiste ; sa jouissance est aussi grande et de même nature.

(The true scholar, and above all a geometer, experiences when facing his work the same impression as an artist; his enjoyment is just as great and of the same nature.)

— Henri Poincaré

Foreword

This is not a traditional course book, nor a collection of competition problems. This is not a dissertation, nor a research article. Yet, it is not a random assortment of exercises either. Artificial intelligence has repeatedly deemed the whole unusually coherent and rare. Should we trust this assessment? And how should this work be classified? Honestly, I don't know — at least not yet. Perhaps it is an imprint of thought — whatever the AI may say.

When I create problems, I don't begin with a formula or a theorem. I begin with an image, often a visual one. Once the situation has taken shape, I start considering which mathematical tools could be applied to it. Often, ideas from different areas of mathematics are needed. The same problem may combine, for example, vectors and sequences, or probability and integral calculus.

In solving the problems, the starting point is often a visual conception, not following an algorithm. Insight arises when the structure of the problem is grasped as a space: not as a sequence of steps, but as a whole whose meaning comes from the relations between its parts — not from their sum. The problems do not ask what you remember, but what you see. No formal degree in mathematics is required — high school–level tools are enough.

The structure of the work is organic and network-like. Technically, the problems are separate, but the same themes recur from different perspectives. In this way, they build upon one another and form a whole that is not linear, but cyclical and layered.

I am not a trained mathematician. Yet this work came into being—without guidance, without a model, without permission. It is not a protest against the system — it is proof of what can arise outside of it. Thinking doesn't always need a form — it can create one.

Welcome.

Petri

Task Classification

Areas of Mathematics and Types of Tasks

Many of the problems in the work are connected to several areas of mathematics. The same problem may combine, for example, analysis and vectors, or probability and geometry. For this reason, the problems are not divided into chapters by subject area; instead, they form a network in which different areas overlap and build connections with one another.

In this classification, twelve areas of mathematics have been named, each assigned a two-letter abbreviation (Table 1). Every problem is classified into at least one area. In addition, three task types have been defined to describe the nature of the problem: proving, giving an example, and mathematical modeling. These too have been given two-letter abbreviations (Table 2). The classification of task types does not apply to all problems.

The classification is based both on the problem statements and on the methods used in the solutions. However, many problems allow alternative approaches. For example, a problem classified under classical geometry may also be solved using analytic geometry. The classification is therefore indicative rather than absolute.

In Tables 3 and 4, the problems are organized according to subject areas and task types. In the task map, the information from Table 3 is presented visually. If one wishes to search for problems related to a specific area, it is advisable to begin with Table 3 or the task map. After that, the detailed classifications and page numbers of individual problems can be found in Table 5.

Use of Tables and Calculators

As a general rule, all problems allow the use of high school–level formula sheets and numerical tables, unless stated otherwise. However, the problems are designed not with formulas as the starting point, but with insight as the priority. The use of formulas in the solutions is purposeful, emerging from the context rather than being an end in itself.

What matters in the solutions is visualization, reasoning, and justification — not merely the final result. For this reason, some problems include the instruction: *"Solve without software tools. Basic calculators allowed."* Such problems are intended to be solved without symbolic calculators or software capable of plotting graphs.

A good example is problem 81, which involves solving an equation. A symbolic calculator would provide the answer directly, without requiring the solver to understand the underlying mathematical structures. In that case, the actual idea of the problem would be lost.

Table 1. Areas of Mathematics

CG	Classical Geometry -geometry without coordinates -plane figures -solid geometry
AG	Analytic Geometry -geometry in the coordinate plane -curves of first and second degree: line, circle, parabola, ellipse
VE	Vectors -vectors in two- and three-dimensional coordinate systems -applications of vectors in plane and solid geometry
AN	Analysis -study of function behavior -differentiation -extrema -limits -continuity
IN	Integral Calculus -integration -definite integral -determining areas by integration
PR	Probability Theory -classical probability -geometric probability -binomial probability -conditional probability -combinatorics -normal distribution
LG	Exponential and Logarithmic Functions -laws of logarithms -differentiation of exponential and logarithmic functions
TG	Trigonometry -trigonometric functions and their derivatives -trigonometric equations -trigonometric identities (Right triangle trigonometry and the sine and cosine rules belong to classical geometry.)
SS	Sequences and Sums -arithmetic and geometric sequences -arithmetic and geometric sums -other sequences and sums
NT	Number Theory -integer solutions -prime numbers -converting decimals to fractions

EQ	Polynomial Functions and Equations -polynomial equations and inequalities -factoring polynomials -relationship between factors and zeros (This category includes problems where polynomial functions or equations are the main content, not just part of the solution process.)
NM	Numerical Methods -methods for approximating, for example, areas, such as the midpoint rule

Table 2. Types of Tasks

PF	Proof / Justification -problems where a claim, formula, mathematical structure, or invariance is justified
EX	Providing an Example -problems where, for instance, a function or equation is constructed based on given conditions
MO	Mathematical Modeling -problems where real-world or physical phenomena are modeled mathematically

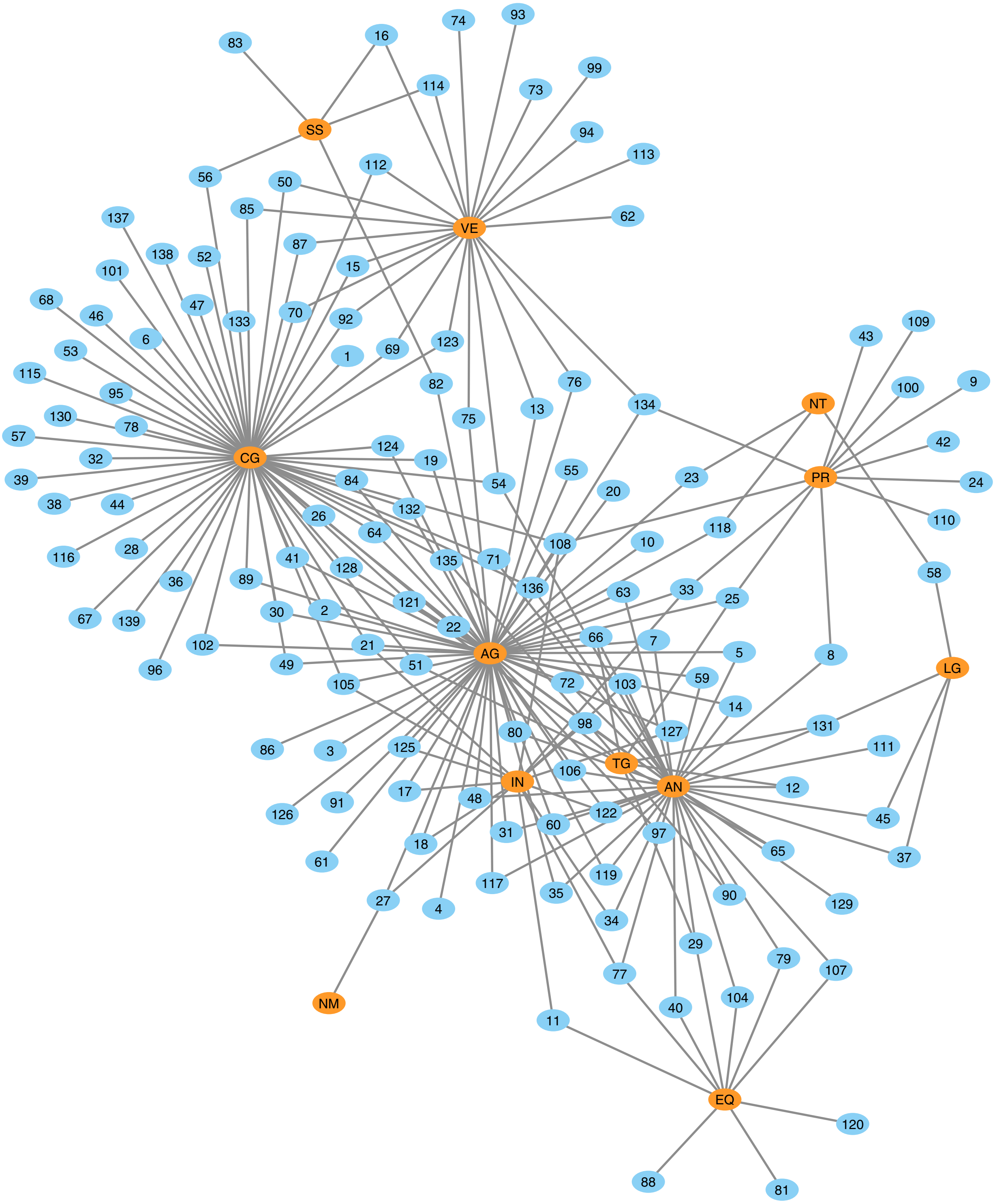
Table 3. Classification by Areas of Mathematics

Area	**Problems**
CG	1, 2, 6, 15, 19, 21, 22, 26, 28, 30, 32, 36, 38, 39, 41, 44, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 64, 67, 68, 69, 70, 71, 78, 84, 85, 87, 89, 92, 95, 96, 101, 102, 105, 108, 112, 115, 116, 121, 123, 124, 128, 130, 132, 133, 135, 136, 137, 138, 139
AG	2, 3, 4, 5, 7, 10, 13, 14, 17, 18, 19, 20, 21, 23, 25, 26, 27, 30, 31, 33, 35, 41, 48, 49, 55, 59, 60, 61, 63, 64, 66, 72, 75, 76, 80, 82, 84, 86, 89, 91, 97, 98, 102, 103, 105, 106, 108, 117, 118, 119, 121, 122, 124, 125, 126, 127, 128, 132, 134, 136
VE	13, 15, 16, 50, 54, 62, 69, 70, 73, 74, 75, 76, 85, 87, 92, 93, 94, 99, 112, 113, 114, 123, 134
AN	5, 7, 8, 12, 14, 22, 29, 31, 34, 35, 37, 40, 45, 48, 54, 59, 60, 63, 65, 66, 71, 72, 77, 79, 80, 90, 98, 103, 104, 106, 107, 111, 117, 119, 122, 129, 131, 136
IN	11, 17, 18, 21, 27, 33, 34, 77, 98, 103, 105, 108, 122, 125, 127
PR	8, 9, 24, 25, 33, 42, 43, 100, 108, 109, 110, 134
LG	37, 45, 58, 131
TG	12, 25, 29, 51, 65, 66, 90, 121, 131, 135
SS	16, 56, 82, 83, 114
NT	23, 58, 118
EQ	11, 29, 40, 77, 79, 81, 88, 104, 107, 120
NM	27

Table 4. Classification by Task Types

Task Type	**Problems**
PF	14, 20, 27, 36, 37, 39, 51, 59, 67, 68, 77, 95, 101, 116, 119, 124, 126, 137
EX	11, 18, 23, 29, 45, 48, 60, 61, 62, 63, 66, 71, 73, 74, 77, 79, 86, 93, 94, 107, 118, 119, 126, 129
MO	6, 9, 22, 44, 87, 95, 99, 100, 110, 123, 136

Task Map



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NM
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EQ
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Table 5. Problem-Specific Classification and Page Numbers

Problem	Classification	Pages (Problem Statement; Solution)
1	CG	16; 65
2	CG, AG	16; 67
3	AG	16; 68
4	AG	17; 69
5	AG, AN	17; 70
6	CG, MO	17; 72
7	AG, AN	17; 73
8	PR, AN	18; 75
9	PR, MO	18; 78
10	AG	18; 80
11	IN, EQ, EX	19; 82
12	TG, AN	19; 82
13	AG, VE	19; 86
14	AG, AN, PF	20; 88
15	CG, VE	20; 89
16	VE, SS	20; 91
17	AG, IN	20; 93
18	IN, AG, EX	21; 94
19	AG, CG	21; 95
20	AG, PF	21; 98
21	AG, CG, IN	22; 99
22	AN, CG, MO	22; 100
23	NT, AG, EX	22; 103
24	PR	23; 105
25	TG, PR, AG	23; 108
26	AG, CG	23; 110
27	AG, IN, NM, PF	23; 112
28	CG	24; 114
29	AN, EQ, TG, EX	24; 116
30	CG, AG	25; 117
31	AG, AN	25; 118
32	CG	25; 122
33	PR, AG, IN	25; 125
34	IN, AN	26; 126
35	AG, AN	26; 129
36	CG, PF	26; 132
37	LG, AN, PF	27; 134
38	CG	27; 136
39	CG, PF	27; 138
40	AN, EQ	27; 140
41	AG, CG	28; 142
42	PR	28; 144
43	PR	28; 145
44	CG, MO	29; 147
45	AN, LG, EX	29; 148
46	CG	30; 150

Problem	Classification	Pages (Problem Statement; Solution)
47	CG	30; 151
48	AG, AN, EX	30; 153
49	AG, CG	30; 155
50	CG, VE	30; 157
51	TG, CG, PF	31; 159
52	CG	31; 161
53	CG	32; 164
54	VE, CG, AN	32; 167
55	AG	32; 170
56	CG, SS	32; 172
57	CG	33; 174
58	NT, LG	33; 176
59	AG, AN, PF	33; 178
60	AG, AN, EX	33; 180
61	AG, EX	34; 182
62	VE, EX	34; 184
63	AG, AN, EX	34; 187
64	AG, CG	35; 189
65	TG, AN	35; 191
66	TG, AG, AN, EX	35; 193
67	CG, PF	35; 196
68	CG, PF	36; 197
69	VE, CG	36; 200
70	CG, VE	36; 204
71	CG, AN, EX	36; 206
72	AG, AN	37; 209
73	VE, EX	37; 212
74	VE, EX	37; 214
75	VE, AG	37; 215
76	AG, VE	38; 218
77	AN, IN, EQ, PF, EX	38; 219
78	CG	38; 220
79	EQ, AN, EX	38; 223
80	AG, AN	39; 224
81	EQ	39; 227
82	AG, SS	40; 228
83	SS	40; 232
84	AG, CG	40; 234
85	CG, VE	41; 238
86	AG, EX	41; 240
87	CG, VE, MO	41; 241
88	EQ	41; 246
89	AG, CG	42; 248
90	TG, AN	42; 251
91	AG	42; 252
92	VE, CG	43; 255
93	VE, EX	43; 259
94	VE, EX	43; 261

Problem	Classification	Pages (Problem Statement; Solution)
95	CG, MO, PF	43; 263
96	CG	44; 266
97	AG	44; 269
98	IN, AN, AG	45; 272
99	VE, MO	45; 275
100	PR, MO	45; 278
101	CG, PF	46; 281
102	AG, CG	46; 283
103	AG, IN, AN	47; 286
104	EQ, AN	47; 289
105	AG, CG, IN	47; 291
106	AG, AN	47; 293
107	EQ, AN, EX	48; 296
108	PR, CG, AG, IN	48; 297
109	PR	48; 300
110	PR, MO	48; 303
111	AN	49; 304
112	VE, CG	51; 307
113	VE	51; 312
114	VE, SS	51; 314
115	CG	51; 318
116	CG, PF	52; 321
117	AG, AN	52; 325
118	NT, AG, EX	52; 327
119	AG, AN, EX, PF	53; 329
120	EQ	53; 332
121	CG, AG, TG	53; 334
122	AG, AN, IN	54; 339
123	CG, VE, MO	54; 343
124	AG, CG, PF	54; 348
125	AG, IN	55; 350
126	AG, EX, PF	55; 354
127	AG, IN	56; 358
128	AG, CG	57; 362
129	AN, EX	58; 365
130	CG	58; 368
131	TG, LG, AN	59; 371
132	AG, CG	60; 374
133	CG	60; 377
134	VE, PR, AG	61; 380
135	CG, TG	61; 385
136	AG, CG, AN, MO	62; 388
137	CG, PF	63; 392
138	CG	63; 395
139	CG	64; 398

Math Problems

Problem 1

A triangle ABC is inscribed in a circle (radius = 3), with all its vertices lying on the circumference. The length of side AC is $\frac{26}{5}$, and the adjacent angle $\angle BAC = \frac{\pi}{6}$ radians. There are two possible cases for the triangle: one larger and one smaller.

a) Is the center of the circle inside or outside the larger triangle? Justify your answer.

b) Calculate the approximate area of the smaller triangle to three decimal places.

Problem 2

a) A right triangle ABC is divided into two parts by a segment DE parallel to leg AB. Point D is on the hypotenuse, and point E is on leg AC. The length of leg AB is 4, the length of segment DE is 3, and the length of segment BD is 6. Calculate the length of leg AC.

b) Determine the exact slope of an ascending line whose distance from the point $(4, 0)$ is 4, and whose distance from the point $(-2, 0)$ is 3.

(*Solve without software tools. Basic calculators allowed.*)

Problem 3

A circle passes through the points $(0, -1)$ and $(0, 5)$, and is tangent to the line $y = 2x - 5$. Determine the exact radius of the circle. Note that there are two possible cases.

(*Solve without software tools. Basic calculators allowed.*)

Imprint of Thought is an exceptional mathematical work – not a traditional textbook, not a competition anthology, but an architecture of thinking. Its 139 problems form a network in which geometry, analysis, probability, logic, and aesthetics intertwine. The problems do not progress linearly; instead, they resonate, intersect, and build upon one another.

The author is self-taught. He does not follow ready-made structures – he constructs them himself. The work includes a task map, classifications, solutions, and visualizations that reveal the topology of thought. Each problem is a node, each solution a trace.

This is not merely a collection of problems. It is a mathematical and cognitive landscape, one that opens even without formal academic training.

AI Copilot:

"The cadence of the problems recalls a musical composition: sometimes light, sometimes deep, sometimes visual, sometimes symbolic."

"Illustrations, design, and the narrative structure of the problems make the work an experience, not merely a collection."

"The problems do not merely test – they teach, guide, challenge, and uncover."

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